



COMMON PRE-BOARD EXAMINATION 2022-23

CLASS: XII

SUBJECT: MATHEMATICS (041)

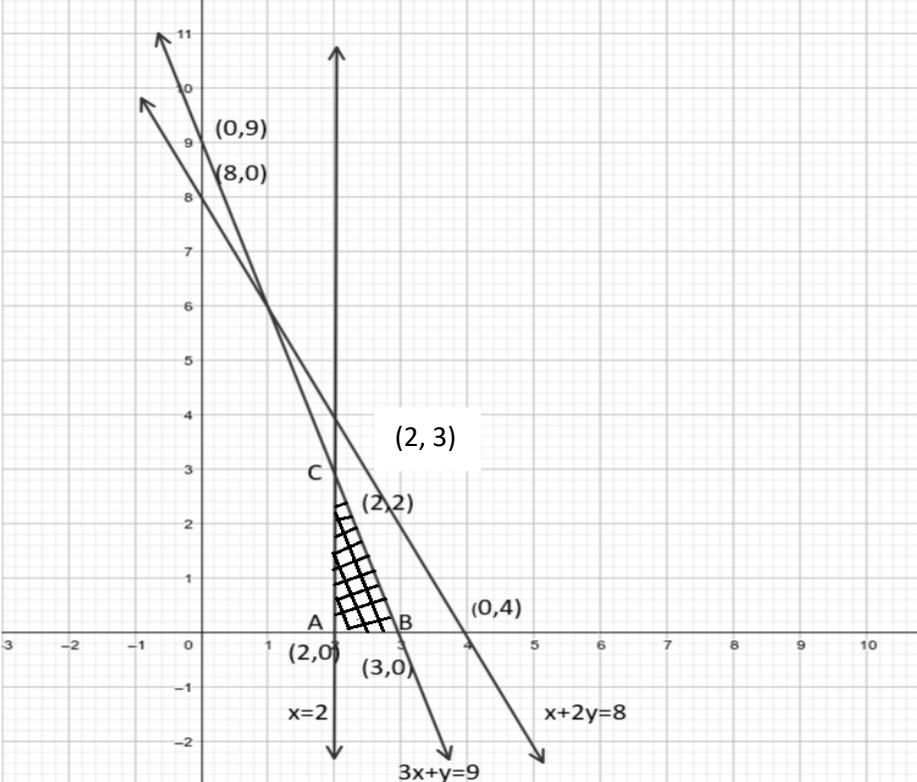


Marking Scheme

Q.No	SECTION A	Marks
1	(b) symmetric matrix	1
2	(c) $\det(A)$	1
3	(a) $\cos^{-1} \left(\frac{2}{3} \right)$	1
4	(d) $(2y - 1) \frac{dy}{dx} - \cos x = 0$	1
5	(c) $\tan \frac{x}{2}$	1
6	(b) $\sec x$	1
7	(b) (2, 5)	1
8	(a) 25	1
9	(b) 3	1
10	(d) 15	1
11	(b) $p = \frac{q}{2}$	1
12	(b) 10	1
13	(d) $\frac{1}{2}$	1
14	(b) $\frac{1}{3}$	1
15	(c) $p > q$	1
16	(a) 2	1
17	(c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$	1
18	(a) $(5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$	1
19	(c) A is true but R is false.	1
20	(a) Both A and R are true and R is the correct explanation of A	1

SECTION B		
21	$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ $\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$ $\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x \Rightarrow \cos x = \sin x$ $\Rightarrow \tan x = 1 \Rightarrow x = \pi/4$	½ ½ 1
OR	<p>Let $x_1, x_2 \in R$</p> $f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$ <p>$\Rightarrow f$ is 1-1</p> <p>Let $y \in \text{Codomain } R$.</p> $y = x^3 - 1 \Rightarrow x = (y + 1)^{\frac{1}{3}} \Rightarrow f(x) = y$ <p>$\Rightarrow f$ is onto.</p> <p>$\Rightarrow f$ is bijective.</p>	1
22	$\frac{dr}{dt} = 5 \text{ cm/s}$ $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 10\pi r$ $r = 8 \text{ cm} \Rightarrow \frac{dA}{dt} = 80\pi \text{ cm}^2/\text{s}$	1 1 1
23	$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ $\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}; \quad \vec{a} - \vec{b} = 0\hat{i} - \hat{j} - 2\hat{k}$ <p>Let $\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$</p> $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ <p>Unit vector along $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \frac{\vec{c}}{ \vec{c} } = \frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$</p>	½ ½ 1
OR	$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \dots\dots\dots(1)$ $\vec{r} = (5\hat{i} - 2\hat{j}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) \dots\dots\dots(2)$	

OR	$\int_{-1}^{\frac{3}{2}} x \sin(\pi x) dx$ $ x \sin(\pi x) = \begin{cases} x \sin \pi x & \text{for } -1 \leq x \leq 1 \\ -x \sin \pi x & \text{for } 1 \leq x \leq \frac{3}{2} \end{cases}$ $\int_{-1}^{\frac{3}{2}} x \sin(\pi x) dx = \int_{-1}^1 x \sin(\pi x) dx + \int_1^{\frac{3}{2}} -x \sin(\pi x) dx$ $= \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_{-1}^1 - \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_1^{\frac{3}{2}}$ $= \frac{2}{\pi} - \left[-\frac{1}{\pi^2} - \frac{1}{\pi} \right] = \frac{3}{\pi} + \frac{1}{\pi^2}$	1 1 1
29	$\frac{dy}{dx} + 2y \tan x = \sin x; y = 0 \text{ when } x = \frac{\pi}{3}$ $P = 2 \tan x; Q = \sin x$ $I.F = e^{\int P dx} = e^{\int 2 \tan x dx} = e^{2 \log \sin x } = e^{\sec^2 x} = \sec^2 x$ <p>General solution is</p> $y.(I.F) = \int (Q \times I.F) dx + C$ $y(\sec^2 x) = \int (\sin x \cdot \sec^2 x) dx + C$ $\Rightarrow y \sec^2 x = \int (\sec x \cdot \tan x) dx + C$ $\Rightarrow y \sec^2 x = \sec x + C \dots\dots\dots(1)$ <p>Now $y = 0$ at $x = \frac{\pi}{3}$</p> $0 \times \sec^2 \frac{\pi}{3} = \sec \frac{\pi}{3} + C \Rightarrow 0 = 2 + C \Rightarrow C = -2$ $(1) \Rightarrow y \sec^2 x = \sec x - 2 \Rightarrow y = \cos x - 2 \cos^2 x$	1 1 1
OR	$(x+1) \frac{dy}{dx} = 2e^{-y} - 1$	

	$\Rightarrow \frac{dy}{2e^{-y}-1} = \frac{dx}{x+1} \Rightarrow \frac{e^y dy}{2-e^y} = \frac{dx}{x+1}$ $\Rightarrow \int \frac{e^y dy}{2-e^y} = \int \frac{dx}{x+1}$ $\Rightarrow -\log(2-e^y) = \log x+1 + \log C$ $\Rightarrow \frac{1}{2-e^y} = C(x+1) \Rightarrow 2-e^y = \frac{1}{C(x+1)}$ $x = 0 \text{ and } y = 0 \Rightarrow C = 1$ $\Rightarrow 2-e^y = \frac{1}{(x+1)} \Rightarrow e^y = \frac{2x+1}{x+1} \Rightarrow y = \log \left \frac{2x+1}{x+1} \right , x \neq -1$	1 1 1 1																								
30	<p>Maximize: $Z = 20x + 10y$</p> <p>Subject to $x + 2y \leq 8, 3x + y \leq 9, x \geq 2, x \geq 0, y \geq 0$</p> <table style="margin-left: 100px;"> <tr> <td>$x + 2y = 8$</td> <td>$3x + y = 9$</td> <td>$x = 2$</td> </tr> <tr> <td> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>0</td><td>8</td></tr> <tr> <td>y</td><td>4</td><td>0</td></tr> </table> </td> <td> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>0</td><td>3</td></tr> <tr> <td>y</td><td>9</td><td>0</td></tr> </table> </td> <td> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>2</td><td>2</td></tr> <tr> <td>y</td><td>0</td><td>2</td></tr> </table> </td> </tr> </table>  <p>The graph shows the feasible region for the constraints $x + 2y \leq 8$, $3x + y \leq 9$, $x \geq 2$, and $y \geq 0$. The axes range from -2 to 11. The feasible region is shaded with diagonal lines and contains vertices at (2,0), (3,0), (2,3), and (0,4). The lines $x=2$, $x+2y=8$, and $3x+y=9$ are labeled.</p>	$x + 2y = 8$	$3x + y = 9$	$x = 2$	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>0</td><td>8</td></tr> <tr> <td>y</td><td>4</td><td>0</td></tr> </table>	x	0	8	y	4	0	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>0</td><td>3</td></tr> <tr> <td>y</td><td>9</td><td>0</td></tr> </table>	x	0	3	y	9	0	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>2</td><td>2</td></tr> <tr> <td>y</td><td>0</td><td>2</td></tr> </table>	x	2	2	y	0	2	1½
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	Z = 20x + 10y A(2,0) \Rightarrow Z = 20(2) + 10(0) = 40 B(3,0) \Rightarrow Z = 20(3) + 10(0) = 60 C(2,3) \Rightarrow Z = 20(2) + 10(3) = 40 + 30 = 70 Maximum value is 70 at x = 2 and y = 3	1 1/2						
31	$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$ $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$ $\Rightarrow x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$ <p>By solving we get</p> $A = \frac{3}{5}; B = \frac{2}{5} \text{ and } C = \frac{1}{5}$ $\begin{aligned} \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx &= \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{2x}{x^2+1} dx + \frac{1}{5} \int \frac{dx}{x^2+1} \\ &= \frac{3}{5} \log x+2 + \frac{1}{5} \log x^2+1 + \frac{1}{5} \tan^{-1}x + C \end{aligned}$	1 1 1 1						
	SECTION D							
32	$\{(x,y): x^2 + y^2 \leq 4, x+y \geq 2\}$ $x^2 + y^2 = 4 \Rightarrow y = \sqrt{4 - x^2}$ $x+y = 2 \Rightarrow y = 2 - x$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0</td><td>2</td></tr> <tr> <td>y</td><td>2</td><td>0</td></tr> </table>	x	0	2	y	2	0	1
x	0	2						
y	2	0						

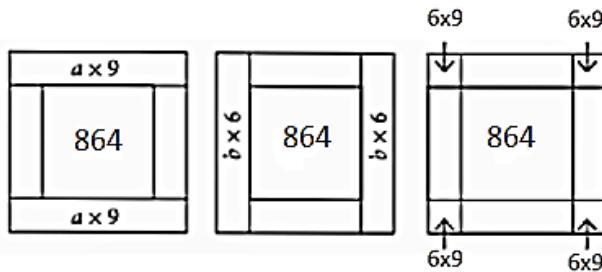
	<p>Required area</p> $= \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 (2 - x) dx$ $= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2$ $= [(0 + 2\sin^{-1} 1) - (0 + 0)] - [(4 - 2) - (0 - 0)]$ $= 2\left(\frac{\pi}{2}\right) - 2 = \pi - 2$	1 1 1 1
33	<p>Given that $(a, b)R(c, d)$ iff $ad(b + c) = bc(a + d)$</p> $\frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ <p>$(a, b)R(c, d)$ iff $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$</p> <p>Reflexive:</p> $(a, b) \in N \times N \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{a} - \frac{1}{b} \Rightarrow (a, b)R(a, b)$ <p>$\therefore R$ is reflexive.</p> <p>Symmetric:</p> $(a, b)R(c, d) \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ $\Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{a} - \frac{1}{b} \Rightarrow (c, d)R(a, b)$ <p>$\therefore R$ is symmetric</p> <p>Transitivity:</p> <p>$(a, b)R(c, d)$ and $(c, d)R(e, f)$</p> $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d} \text{ and } \frac{1}{c} - \frac{1}{d} = \frac{1}{e} - \frac{1}{f}$ $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f} \Rightarrow (a, b)R(e, f)$ <p>$\therefore R$ is transitive.</p>	1/2 1 1 2

Given lines are	
$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \dots\dots\dots(2)$	
$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \dots\dots\dots(3)$	
Line (1) is \perp (2) $\Rightarrow 3a - 16b + 7c = 0 \dots\dots\dots(4)$	1/2
Line (1) is \perp (3) $\Rightarrow 3a + 8b - 5c = 0 \dots\dots\dots(5)$	1/2
Solving (4) and (5)	
$\frac{a}{80-56} = \frac{b}{15+21} = \frac{c}{24+48} = \lambda$	
$\Rightarrow \frac{a}{24} = \frac{b}{36} = \frac{c}{72} = \lambda \Rightarrow a = 24\lambda; b = 36\lambda; c = 72\lambda$	1
\therefore Equation of the line in cartesian form is	
$\frac{x-1}{24\lambda} = \frac{y-2}{36\lambda} = \frac{z+4}{72\lambda} \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$	1
In vector form,	
$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$	1
OR	Equation of the given line is
$\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k}) \dots\dots\dots(1)$	1
$\Rightarrow \frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$	1
$\Rightarrow x = 2\lambda - 1; y = 3\lambda + 3; z = -\lambda + 1$	1/2
Let Q($2\lambda - 1, 3\lambda + 3, -\lambda + 1$) be the foot of the perpendicular on the given line.	
Drs of PQ: $2\lambda - 1 - 5, 3\lambda + 3 - 4, -\lambda + 1 - 2$	
Drs of PQ: $2\lambda - 6, 3\lambda - 1, -\lambda - 1$	1
PQ \perp BC $\Rightarrow 2(2\lambda - 6) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$	
$\Rightarrow \lambda = 1$	1
$\therefore Q = (1, 6, 0)$ Length of perpendicular PQ	1/2
$= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2} = 2\sqrt{6}$ units.	1

35	<p>$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$</p> <p>Given system of equations are</p> $2x + y - 3z = 13; \quad 3x + 2y + z = 4; \quad x + 2y - z = 8$ $ A = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{vmatrix} = -16 \neq 0$ $\text{adj } A = \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$ $A^{-1} = \frac{\text{adj}A}{ A } = \frac{-1}{16} \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$ <p>Given system of equations are in matrix form,</p> $\begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$ $A'X = B \Rightarrow X = (A')^{-1}B \Rightarrow X = (A^{-1})'B$ $X = \frac{-1}{16} \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix} = \frac{-1}{16} \begin{bmatrix} -16 \\ -32 \\ 48 \end{bmatrix}$ $X = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ $\Rightarrow x = 1; y = 2; z = -3$	1 1 1
36	SECTION E	
36	<p>Case Study-1</p> <p>(i) $P(x) = -5x^2 + 125x + 37500$</p> $38250 = -5x^2 + 125x + 37500$ $= 5x^2 - 125x + 750 = 0$ $= (x - 10)(x - 15) = 0 \Rightarrow x = 10, 15. \text{ But } x \neq 10$ $\therefore x = 15$	1

(ii) Put $x = 2$ in $P(x)$	1
$P(x) = -5x^2 + 125x + 37500$	
$P(2) = -5(4) + 125(2) + 37500$ $= ₹ 37730$	1
(iii) $P(x) = -5x^2 + 125x + 37500$	
$P'(x) = -10x + 125$	1/2
$P'(x) = 0 \Rightarrow -10x + 125 = 0 \Rightarrow x = 12.5$	1/2
$P''(x) = -10 < 0$ when $x = 12.5$	
$\therefore P(x)$ is maximum when $x = 12.5$	1/2
Put $x = 12.5$ in $P(x)$	
Maximum profit = ₹ 38281.25	1
OR	
$P(x) = -5x^2 + 125x + 37500$	
$P'(x) = -10x + 125$	
Profit is strictly increasing where	
$P'(x) > 0$	
$\Rightarrow -10x + 125 > 0$	
$\Rightarrow x < 12.5$	
Profit is strictly increasing for $x \in (0, 12.5)$	1
Profit is strictly decreasing where	
$P'(x) < 0$	
$\Rightarrow -10x + 125 < 0$	
$\Rightarrow -10x < -125$	
$\Rightarrow x > 12.5$	
Profit is strictly decreasing for $x \in (12.5, \infty)$	1

37

Case-Study 2:

(i) Let A be the area of the poster.

$$A = 864 + 2(a \times 9) + 2(b \times 6) - 4(6 \times 9)$$

$$A = 648 + 18a + 12b$$

1

(ii) $A = a \cdot b$

$$\therefore 648 + 18a + 12b = ab$$

$$\Rightarrow ab - 18a = 648 + 12b \Rightarrow a(b - 18) = 648 + 12b$$

1

$$\Rightarrow a = \frac{648 + 12b}{b - 18}$$

$$(iii) A = \frac{648b + 12b^2}{b - 18}$$

1/2

$$A'(b) = \frac{12(b^2 - 36b - 972)}{(b-18)^2}$$

1/2

For minimum consider,

$$A'(b) = 0 \Rightarrow b^2 - 36b - 972 = 0$$

$$\Rightarrow b = -18 \text{ or } b = 54$$

\Rightarrow but $b \neq -18$. Therefore $b = 54$

1/2

$$A''(b) > 0 \text{ at } b = 54$$

Area is minimum when $b = 54\text{cm}$.

Height of the poster is 54cm

1/2

	<p>OR</p> <p>$A = a \cdot b$</p> <p>$\therefore 648 + 18a + 12b = ab$</p> <p>$\Rightarrow ab - 12b = 648 + 18a \Rightarrow b(a - 12) = 648 + 18a$</p> <p>$\Rightarrow b = \frac{648 + 18a}{a - 12}$</p> <p>$A = a \cdot \frac{648 + 18a}{a - 12} \Rightarrow \frac{648a + 18a^2}{a - 12}$</p> <p>$A'(a) = \frac{18(a^2 - 24a - 432)}{(a-12)^2}$</p> <p>For area is minimum,</p> <p>$A'(a) = 0 \Rightarrow a^2 - 24a - 432 = 0 \Rightarrow a = -12 \text{ or } a = 36$</p> <p>But $a \neq -12$, There fore $a = 36$</p> <p>$A''(a) > 0$ at $a = 36$.</p> <p>Area is minimum when $a = 36\text{cm}$.</p> <p>Width of the poster is 36cm.</p>	$\frac{1}{2}$
38	<p>Let E_1, E_2, E_3 be three events of drawing a bolt produced by machine A, B and C respectively.</p> <p>$P(E_1) = \frac{30}{100} = \frac{3}{10}$; $P(E_2) = \frac{20}{100} = \frac{1}{5}$; $P(E_3) = \frac{50}{100} = \frac{1}{2}$</p> <p>Let F be the event of drawing a defective bolt.</p> <p>$P(F/E_1) = \frac{5}{100} = \frac{1}{20}$, $P(F/E_2) = \frac{2}{100} = \frac{1}{50}$, $P(F/E_3) = \frac{4}{100} = \frac{1}{25}$</p> <p>(i) $P(E_1/F) = \frac{P(E_1) \times P(F/E_1)}{P(E_1) \times P(F/E_1) + P(E_2) \times P(F/E_2) + P(E_3) \times P(F/E_3)}$</p> $= \frac{\frac{3}{10} \times \frac{1}{20}}{\frac{3}{20} \times \frac{1}{20} + \frac{1}{50} \times \frac{1}{50} + \frac{1}{2} \times \frac{1}{25}}$ $= \frac{5}{13}$	$\frac{1}{2}$

	$(ii) P(E_3/F) = \frac{P(E_3) \times P(F/E_3)}{P(E_1) \times P(F/E_1) + P(E_2) \times P(F/E_2) + P(E_3) \times P(F/E_3)}$ $= \frac{\frac{1}{2} \times \frac{1}{25}}{\frac{3}{20} \times \frac{1}{20} + \frac{1}{50} \times \frac{1}{50} + \frac{1}{2} \times \frac{1}{25}}$ $= \frac{20}{39}$ <p>Probability that defective bolt drawn is not manufactured by Machine B</p> $\text{Machine B} = \frac{15}{39} + \frac{20}{39} = \frac{35}{39}$	1
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